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Anticipating the Future

Leonardo's Unpublished Anatomical and Mathematical Observations

Morteza Gharib, Francis C. Wells, with Mari-Tere Álvarez

I flung myself into futurity.
—H. G. WELLS, 1895

In 1895, H. G. Wells penned these words in his science-fiction novel *The Time Machine.* For Wells and for Early Modern writers, such as Thomas More (1478–1535), Francis Bacon (1561–1626) (discussed in other chapters), and Margaret Cavendish (1623–1673), publishing science fiction permitted the imaginings of a future, with new ideas and new technology. To these futurists, futurity was not just the future but rather a state of mind, an awareness of the still-to-come, the potential and excess of the not-yet-here. Subjects actively oriented themselves toward this utopic future.

This mindset does not describe fifteenth-century polymath and artist Leonardo da Vinci (1452–1519), despite popular understanding of him as a prescient genius. Leonardo's assumed implausible ideas were misunderstood in the Renaissance and came to fruition only in the modern age as a latent futurity. This essay examines Leonardo's drawings and texts on two seemingly fantastical subjects in the Renaissance—the anatomy of the human heart and his musings on velocity—both instances in which the artist and genius seems to predict the future. In the conclusion, we return to the differences between a fifteenth-century inventor and a futurist writer.

LEONARDO AND THE HUMAN HEART

Leonardo's interest and involvement in anatomical study appears to have begun at a very early stage and continued throughout his life. His extant notes reveal a progression in his thinking, from a conception of the human heart based on Galenic representations to, in his later years, more original and abstract thinking. His later observations on the human heart, in particular, demonstrate an experimenter in search of knowledge as opposed to an inventor predicting future needs in the field of cardiology three hundred years in the future. In other words, Leonardo's findings were of no applied use for physicians of the time. For example, his descriptions of functional anatomy, including the closure mechanism of the aortopulmonary valves, had no practical application at the time. It would be an additional 150 years before the English physician William Harvey made his seminal contribution to the history of medicine by describing the systemic circulation of blood through the human body.[1] Consequently, there were no therapies for heart-valve disease in Leonardo's time. Indeed, the effect of diseases such as rheumatic heart-valve disease, perhaps the most common affliction of the time, is not mentioned in then-current medical texts and pharmacopeias.[2] Furthermore, based on the evidence of his notebooks, there does not seem to be direct evidence that Leonardo had meaningful relationships with the physicians of his day.[3]

Without a doubt, however, Leonardo's drawings and descriptions of the workings of the heart valves coupled with his visual descriptions of the presence of a dual blood supply to the lungs are strikingly prescient and speak to a distant future. With that said, however, it is important to emphasize that Leonardo was not a futurist, but rather was deeply grounded in the present. Evidence of his mindset includes the fact that he did not bother to publish his findings, which, in fact, were not published until the late nineteenth century, in Sabachnikov's two-volume work reproducing the Windsor anatomical collection.[4] Nevertheless, Leonardo's brilliant deductions about the physiognomy of the human heart—based on his close and accurate observation allied to engineering knowledge, and when combined with the fact that the drawings are still considered valid and accurate descriptions of the heart today—only strengthen Leonardo's icon status to writers and readers of science fiction.[5]

CORONARY ATHEROSCLEROSIS

Leonardo's observations about coronary atherosclerosis present another case where contemporary scholars see a futurist Leonardo. According to his notebooks, in the winter of 1507–8, Leonardo spent time at the local hospital (Santa Maria Nuova) in Florence with a patient who claimed to be one hundred years old. Leonardo recounted how the old man died peacefully in front of him. Intrigued by what he described as a "death so sweet," Leonardo stated that he "made an anatomy," what we would now call an autopsy, in an attempt to understand the cause of this quiet death.[6] This story is confirmed by a contemporary source (Anonimo Gaddiano), who described Leonardo as making a series of noteworthy drawings: "all marvelous things and among them one of our Lady and a Saint Anne, which went to France, and anatomical studies which he drew in the Santa Maria Nuova in Florence."[7]

The dissection of the old man would appear to have been very comprehensive; Leonardo left extensive drawings of the cadaver's upper body and refers in his handwritten notes to the portal venous system and the arterial supply to the small intestine.[8] The most unique information from this particular autopsy is the extensive, detailed description that Leonardo made about the state of the heart; from this description he made a number of important deductions. Through a series of drawings and notes he reported the thickening and narrowing of the coronary arteries and appears to have drawn the correct conclusion that the cause of death was the restriction of blood to the muscle of the heart. This appears to be one of the earliest descriptions of what is known as hardening of the arteries or coronary atherosclerosis, which he then identifies as the cause of death. Leonardo even goes on to issue a health warning suggesting that this process may be attributed to a lack of exercise. Clearly intrigued by this finding, Leonardo then reported the need to study the nature of arteries in the young and old as well as do comparative anatomy on "birds of the air and animals of the field."[9] He concluded that increased thickening and tortuosity of the vessels are significant parts of the aging process.

Unique and important though this description of atherosclerosis is, Leonardo's seemingly futuristic observations on cardiac pathology were not unique. Florentine physician Antonio Benivieni (1443–1502) reported on an autopsy performed on one of his relatives.[10] His documentation of the procedure seems to include one of the earliest descriptions of rheumatic mitral valve stenosis, a condition whereby the narrowed mitral valve does not open properly: "Wherefore the cadaver of the deceased having been cut up for public benefits, it was found that the ventricle of the man was so hardened in the joints of the opening toward its lowest part that since it was able to transmit nothing from there to the inferior parts, by necessity death followed."[11]

THE HEART DISSECTIONS

Without a doubt, Leonardo's examinations of the heart read like modern anatomical medical scans. The heart is a four-chambered muscle pump with two inlet and two outlet unidirectional nonreturn valves. Described thus it may sound like a simple organ whose structure and function should be easy to discern. However, this is far from the truth. Almost all of Leonardo's existing notes and drawings of the ox heart reveal an intense study of the external form of the heart and the related coronary arteries and veins as well as detailed examination of the interstices of the cardiac chambers. A series of small drawings of the heart reveals that Leonardo clearly understood that the heart naturally twists while contracting, otherwise known as cardiac twist, a process that allows the maximum emptying of the cardiac ventricles.[12]

Examination of the heart is most commonly done when it is in the relaxed and nonworking state, both in the postmortem room and in the operating theater.

This flaccid state denies a proper appreciation of its form when it is working and is full of blood and under normal muscular tension. Leonardo appreciated this and described a novel technique to demonstrate the heart in its working state. He injected the chambers of the heart with hot wax under pressure and waited for the wax to set. This gave a cast of the internal shape and, after rigor mortis had developed, caused the heart to set in its working form. Leonardo also used this technique to better understand the ventricles of the brain. Perhaps its best use was in the aortic root at the base of the heart, more of which will be described later. Although he did not publish or disseminate his findings, Leonardo's drawings are incredibly informative, as seen in the case of his drawing of the tricuspid atrioventricular valve.[13] The drawing describes cutting out the valve to demonstrate how the valve works. Both C. D. O'Malley and J. B. de C. M. Saunders rightfully suggest that the drawing is made in such a way that allows the reader to reconstruct the orifice as suggested.[14]

THE MECHANISM OF AORTIC AND MITRAL VALVE CLOSURE

The valves of the heart open and close in response to the flow of blood, which in turn is generated by atrial and ventricular contraction and relaxation (figure 5.1). However, today we understand that the exact mechanism of complete and safe closure is rather more complex and requires vorticeal flow above the aortopulmonary valves, which evenly unfurl the opened leaflets to ensure complete closure. Leonardo discovered this complex mechanism in his later studies while examining ox hearts. The aortic valve sits at the outlet of the left ventricle, where the aorta arises from the base of the heart. The origin of the aorta has distinctive bulges eponymously named the sinuses of Valsalva (the only related contribution in the literature is an engraving in a tractus on the ear by Morgagni with no commentary).[15]

Leonardo explored in detail the reason for this anatomical oddity and was able to explain it in functional (physiological) terms. The coronary arteries arise from two of these sinuses. In the ox heart these are very striking bulges. Applying his knowledge of hydrodynamics, he described the formation of vortices within these sinuses that cause the leaflets of the aortic valve to unfurl from their base and to meet evenly, giving full aortic valve leaflet closure. These vortices arise as the blood expands into the space, having passed through the relatively narrowed orifice of the aortic valve itself. In addition, where the sinuses meet the beginning of the aortic tube, they encounter a relative narrowing. This junction is named the sinotubular junction. It is naturally 10 percent narrower than the opening of the valve, and hence the peripheral blood in the column that is leaving the left ventricle will strike this ridge and be diverted backward along the curved wall of the sinus to the base of the aortic valve leaflet and then along the leaflet as the peripheral blood of

FIGURE 5.1. Leonardo da Vinci, Notes on the valves of the heart and flow of blood within it, with illustrative drawings, ca. 1513, pen and ink on blue paper. Windsor, Royal Library, MS 19082r. Royal Collection Trust © Her Majesty Queen Elizabeth II 2017.

the vortex, which will unfurl the leaflet to give rise to complete leaflet closure. If valve closure relied simply upon the reflux of the column of blood when forward flow ceased, then the leaflets would furl upon themselves, rendering the valve incompetent. Leonardo described all of this and drew a little diagram that very convincingly illustrated his thoughts about this.[16] Moreover, he continued with his thinking, using his knowledge of fluid drag and nonlaminar flow. He drew many diagrams to show the reduced velocity and hence height that any peripheral fluid will reach in a column flowing through a tube as a result of fluid drag giving rise to nonlinear flow.[17] And, although we have these detailed drawings and notes, there is no known dissemination of this profound knowledge.

Leonardo then went even further and wrote a discussion of the importance of the elasticity of the tissues of the aortic root. During ventricular ejection, the aortic root is capable of expanding beyond its natural size at rest to allow maximal ventricular emptying on severe exercise, creating an even more dynamic structure. This elasticity is then converted into a shock-absorbing mechanism upon valve closure.[18] Leonardo correctly ascribed the necessity of this property to prevent avulsion of the aortic valve leaflets closing under high pressure. In our modern world, this property of these structures has real relevance, as the lack of this elasticity in some of the earlier man-made prosthetic tissue valves used in modern heart surgery has resulted in the leaflets tearing away from the supporting stent posts.[19] This potentially life-threatening catastrophic valve failure caused the withdrawal from the marketplace of these valves and a subsequent redesign.

Additionally, Leonardo went on to describe why these valves must have three leaflets, not two or four, using geometry to prove that the three-leaflet option gives the greatest strength and durability.[20] It is a fact that bi-leaflet aortic valves tend to fail before the natural lifespan of the valve is reached and often very early. Patients with bi-leaflet aortic valve abnormalities make up about 10 percent of all patients presenting with aortic valve disease. An interesting aside to these notes by Leonardo is whether or not he ever saw a bi-leaflet valve, since the likelihood of observing such an anomaly in an ox heart is small. Perhaps he found one in a human dissection, or possibly it is more likely that this was a thought game analyzing all options and applying geometry to answer the question. Since Leonardo did not in any way make this knowledge known via publication, none of this work would be of any relevance to any professional in medicine, anatomy, or pathology for the next five hundred years.

THE CLOSURE LINES OF THE HEART VALVE LEAFLETS

It could be assumed that the leaflets of the heart valves close in the way that a door closes within a doorframe, but this is mistaken. For a good seal to occur and to spread the load and shock of closure, the leaflets of the heart valves meet over a vertical distance of approximately 0.7 mm to 1 cm, a closure line that is

FIGURE 5.2. Leonardo da Vinci, Detail of atrioventricular valve leaflets, ca. 1513, pen and ink on blue paper. Windsor, Royal Library, MS 19074r. Royal Collection Trust © Her Majesty Queen Elizabeth II 2017.

called coaptation in the description of valve anatomy. It is another observation of Leonardo that this is essential for normal valve function. This arrangement is beautifully shown in a detailed drawing on Windsor manuscript RL 19074 *recto* for the tricuspid valve and again on Windsor RL19080 *recto* for the mitral valve (figure 5.2).

This anatomical arrangement is essential to reproduce in modern-day mitral and tricuspid valve reconstruction to ensure proper closure and an efficient seal for these valves. The same principle applies for the aortic and pulmonary valves. This is demonstrated without comment in the drawings of the aortopulmonary valves in another manuscript.[21] Leonardo's ability to document tiny aspects of anatomical form in such detail reinforces his own statement of the intensity of his application and observation.[22]

THE MOVEMENT OF THE HEART

Leonardo's understanding of synchronicity and reciprocity of the movement of the atria and the ventricles of the heart is another of his original contributions. This is to be detected in the only drawing of the heart that is to be found outside of the Windsor collection.[23] In fact, Leonardo was the first individual to describe the heart as a four-chambered structure. Before his description, the heart was considered to consist only of the two ventricles. The atria were not considered to be part of the heart. He continued to interchange the terms for the atria as either ventricles or atria but was the first to ascribe both pairs of chambers to the heart.[24] He used the presence of the atrioventricular valves as proof that the atria and ventricles were separate and important independent chambers.[25] In this section he described the important differences in structure, with the atria having thinner walls than the ventricles. In this passage, he also described the alternating contraction and relaxation of the upper (atria) and the lower ventricles.

In addition to the synchronized motion of the cardiac chambers, as mentioned earlier, Leonardo also began a discussion on the rotational movement of the heart in an observational study that he did in Tuscany. While observing the killing of pigs with a *spillo* (metal spike) that was plunged into their hearts through the chest wall, he noticed that, depending on the point of the entry of the spike into the heart, the rotational movement differed; if the spike penetrated the heart in the midpoint of the ventricle, there was no movement, a point of equipoise. He described leaving the instrument in the heart until the pig was cut up to test his observation. He commented that he saw this "many times."[26] This is the earliest description of a key component of heart motion that is known as cardiac twist. It is an important topic in the modern study of heart failure. This twisting action allows the heart to empty more completely than by simple muscular contraction. No heart muscle fiber can shorten by more than 20 percent of its length. By twisting itself in systolic contraction, it enhances emptying in the same way as the wringing out of a wet cloth to extrude water. The relaxed heart is in the shape of a complex cone, a cone with a twist. The filling of the heart with blood untwists it so that when the filling phase ceases there is a moment of force returning the heart toward its original state as contraction commences. This helps to overcome the inertia at the start of cardiac motion. It is striking that Leonardo should be so interested in this complex motion of the heart and so observant that he could identify it, presaging modern physiological thought.

THE DUAL CIRCULATION OF THE LUNGS

Another of Leonardo's important observations is that the lung has two contributing blood supplies (figure 5.3). We now know that deoxygenated blood exits the heart through the pulmonary arteries to enter the lungs, where the blood's oxygen is replenished as air is drawn into the lungs and as carbon dioxide produced in the metabolic processes is removed; these gases are expelled in the exhaled breath. This gaseous exchange takes place across the membranes of the smallest blood vessels, the capillaries, which are one cell thick, and the single-celled small air sacs, to which the capillaries are intimately applied in the walls of the alveolar sacs. The second source of blood to the lungs is via a series of much smaller vessels called the bronchial arteries. These are hard to find in the dissection room even when the prosector knows that they are there. They arise from the underside of the arch of the aorta and ramify along the main bronchi following their divisions deep into the lungs to their termination. Unlike the pulmonary artery they carry oxygenated

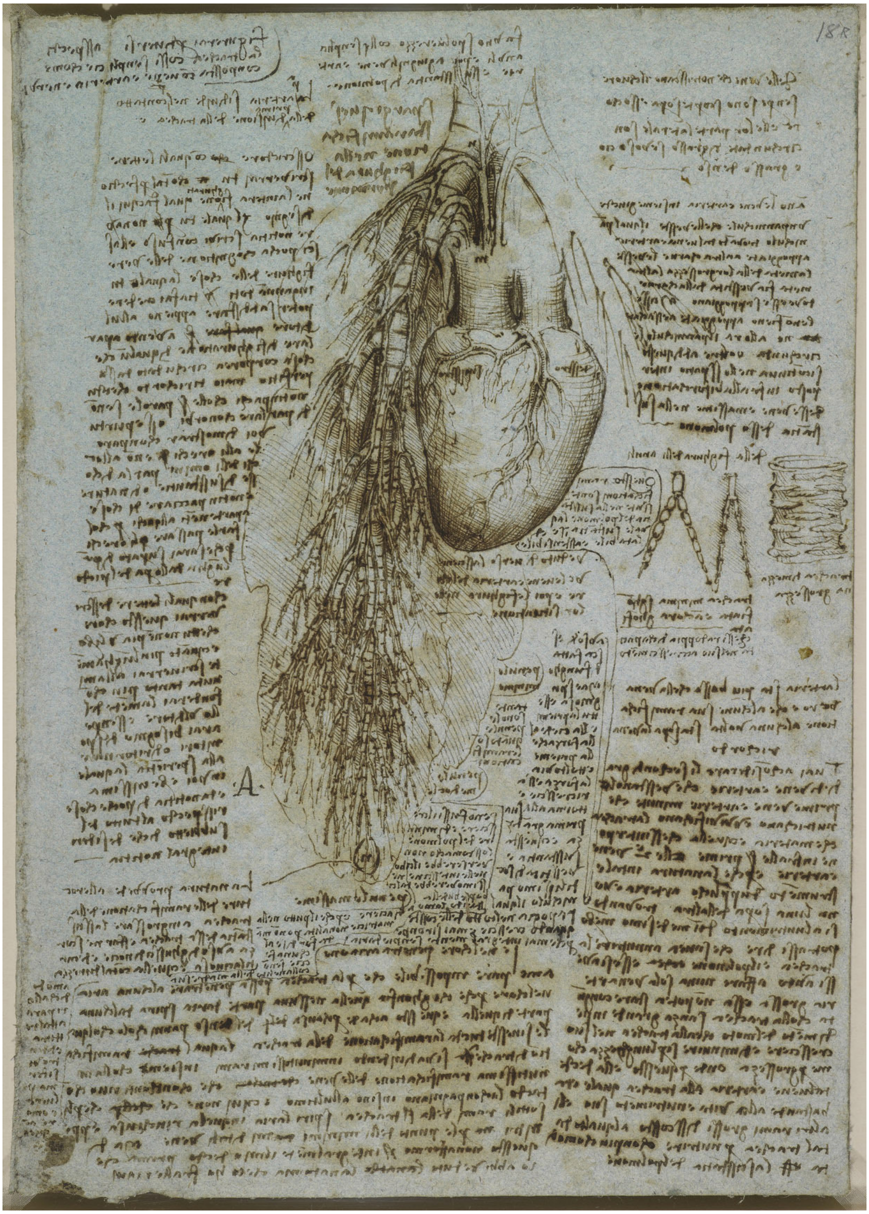

FIGURE 5.3. Leonardo da Vinci, The heart and lungs dissected to reveal the bronchi and the accompanying bronchial arteries, ca. 1513, pen and ink on blue paper. Windsor, Royal Library, MS 19071r. Royal Collection Trust © Her Majesty Queen Elizabeth II 2017.

blood to nourish the major airways. They arise as a result of the embryological development of the lungs.

This detailed understanding of the pulmonary blood supply did not exist in the time of Leonardo who, initially, continued to be influenced by the Galenic description of blood circulation. The physiology of breathing was not at all understood. Galen had postulated that air passed directly across the trachea and from the lungs into the heart and that air and blood mixed directly. As his observational work progressed, Leonardo began to directly challenge Galen's theories. He wrote:

> Whether air penetrates into the heart or not. To me it seems impossible that any air can penetrate into the heart through the trachea, because if one inflates the (lung), no part of the air escapes from any part of it. And this occurs because of the dense membrane with which the entire ramification of the trachea [bronchi] is clothed. This ramification of the trachea as it goes on divides into the minutest branches together with the most minute ramification of the veins which accompany them in continuous contact right to the ends. It is not here that the enclosed air is breathed out through the fine branches of the trachea and penetrates through the pores of the smallest branches of these veins. But concerning this I shall not wholly affirm my first statement until I have seen the dissection which I have in hand.

To challenge Galen at that time was to take on all of the force of the academics. It is this kind of proclamation of individual thought that probably led Leonardo to rail against the academics of the time with the following note:

> Many will think that they can with reason blame me, alleging that my proofs are contrary to the authority of certain men held in great reverence by their inexperienced judgment, not considering that my works are the issue of simple and plain experience which is the true mistress . . . I am fully aware that the fact of my not being a man of letters may cause certain presumptuous persons to think that they may with reason blame me, alleging that I am a man without learning. Foolish folk![27]

As discussed earlier, Leonardo's discovery of the second system of blood vessels, the bronchial arteries, was probably enhanced by the fact that he was dissecting the heart and lung block of an ox, in whom these structures are naturally larger than in a human specimen. Nevertheless, having reproduced the dissections of Leonardo, I have found that even in this setting they are not easy to demonstrate and require a high level of dissection skill. Leonardo's discovery is even more exceptional because no one knew of their existence beforehand. His handwritten notes unmistakably point up the separateness of the vascular systems and the need for them: "You have to consider the second order of veins and arteries which cover the first [i.e., bronchial arteries and veins] minute veins and arteries which nourish and vivify the trachea the substance of which is interposed between the first and second [order] of veins and arteries; and why Nature duplicated artery and vein in such an instrument, one above the other, for the nourishment of one and the same organ." He went on to suggest that the reason for this may be that the main pulmonary artery could not be the source of blood for these airways rather than the lung tissue, as the attachment of those great vessels to the airways would impede the movement of the airways within the lungs, which is often violent extreme exercise. He concluded: "Wherefore this reason she [Nature] gave such veins and arteries to the trachea sufficient for its life and nourishment [the bronchial circulation] and the other big branches she separated somewhat from the trachea in order to nourish the substance of the lung [the parenchyma] more conveniently."[28] These vessels received no mention in Andreas Vesalius's *De humani corporis fabrica* of 1543. Leonardo's physiological reasoning and astute observation are exceptional, predicting future knowledge of the anatomical record.

RECOGNITION OF LEONARDO'S ANATOMICAL ENDEAVORS

Leonardo's anatomical observations would not be recognized for hundreds of years. In fact, the only contemporary acknowledgment of Leonardo's anatomical endeavors is cursory and does not dwell on what was achieved. Antonio de Beatis, secretary to the Cardinal Luigi of Aragon, notes his visit to Leonardo's studio in Amboise on October 10, 1517, and remarks:

> This gentleman has written of anatomy with such detail, showing with illustration the limbs, veins, tendons, intestines and whatever else there is to discuss in the bodies of men and women in a way that has never yet been done. All this we have seen with our eyes; and he said that he had dissected more than thirty bodies of men and women of all age. He has also written on the nature of water, on various machines and on other matters, which he has set down in an infinite number of volumes all in the vulgar tongue; which if they were published would be useful and very delighful.[29]

There could be other reasons why his observations on anatomy were not appreciated. Monica Azzolini points out that Leonardo's lower position in society, owing to his illegitimacy and lack of a classical education (and only rudimentary command of Latin), would put him at a disadvantage in the elite academic societies of Milan, Florence, and Rome.[30] Despite this reasonable challenge to the claim of his mental supremacy over the ages, it cannot be ignored that, aside from the utility of the musculoskeletal anatomy for the surgeon and the mapping of superficial veins for bloodletting, the functional descriptions that Leonardo made of the internal organs were of no use to physicians or surgeons of that era or for many years to come. Indeed, the derived understanding of very complex functional anatomy that Leonardo displays in his later anatomical pages does indeed place him in the stratosphere of intellect across the ages and in these areas; whereas his work might have anticipated the future, he himself was not contemplating it in terms of the future.

MATHEMATICS

The idea for this portion of the essay originated with a sentence from Martin Kemp's book *Seen/Unseen* in a chapter devoted to the art of analogy.[31] Kemp writes about how the use of analogy need not be purely qualitative when scientists apply it to understanding natural processes. He gives the example of Leonardo da Vinci's applying the law of conservation of volume, which he had deduced from his studies of river flows, to explain the way tree branches bifurcate. As Kemp puts it, Leonardo comprehended the unwritten (undiscovered in Leonardo's time) physical law that governs bifurcation of tree branches through similitude and analogy as if he could see this law unformulated before him.[32] Modern scientists rarely find opportunities to experience this process, since they are trained to see nature and the physical processes that govern it through existing equations of physics. Hence, they are limited to seeing only cases that fit as solutions to equations that we choose to use. While powerful as tools for finding solutions, these equations often limit the view of the broader connections that link natural processes. In contrast, Leonardo da Vinci observed nature not only for its naked beauty via his artistic work but also as a manifestation of related and harmonic solutions to unseen and unwritten equations of natural laws. He achieved this understanding by applying qualitative and quantitative analogies. His approach, while limited by the primitive methods of measurements of his time, allowed him to reach a much deeper level of understanding of laws of nature than that of his contemporaries.

This section will examine two examples of Leonardo's qualitative and quantitative approach to using an analogy as a tool to decipher natural laws. One deals with his study of ballistic trajectories, where he envisions a parabolic trajectory despite experimental evidence to the contrary and exact mathematical equations in support of his parabolic postulate. The second example deals with seeing the unseen equations of motion nearly two hundred years before the observations of Isaac Newton.

MOTION

Leonardo was intrigued by the trajectories of projectiles and falling objects. His fascination with mysterious forces of gravity occupied him through many studies on the nature of weight, force, and motion under different static and dynamic conditions. He was familiar with the Medieval concept of "impetus," which influenced objects to gain acceleration toward the earth. He would describe acceleration by noting that "a weight that descends freely in every degree of time acquires . . . a degree of velocity."[33] This statement indicates that Leonardo had correctly understood that the velocity of a falling object has a linear relationship with time, a discovery that is popularly attributed to Galileo. In mathematical notation, we can translate Leonardo's statement to

$$v = gt , \tag{1}$$

where g is a constant factor to be defined by the then-modern physics/mechanics of Newton's time as the local constant of gravity. Obviously, this mathematical language and the concept of "equations" regarding functions and variables were not known to fifteenth-century Leonardo or even to Galileo, who lived 140 years later. For them, the mathematical proof was performed through Euclidean geometry. Galileo's approach was more advanced than Leonardo's, since he added proportionality and ratios of quantities to prove his postulates or describe his observations. Also, unlike Newton, Leonardo did not have access to abstract mathematical methods such as algebra and calculus.

As can be seen in equation (1), which is also considered as one of the equations of motion, the accurate measurement of time is central to any experimental studies of falling objects. In this respect, a major impediment to Leonardo's observations of falling objects was his lack of access to accurate timing instruments. This limitation had naturally forced him and all of his contemporaries to resort to the measurement of distance in representing time. By using distance to represent itself and time concurrently, it would be difficult if not impossible to develop a clear definition for velocity as "the distance traveled over a time interval"; that would as well be true for the definition of acceleration. More accurate timing devices became available only during Galileo's time; equations and calculus were tools of Newton's time.

Two hundred years later, in the seventeenth century, we begin to see equations that describe kinematics of bodies in motion (equations of motion) mainly in the published works of René Descartes, Isaac Newton, and Gottfried Leibniz. These equations can be derived from the basic definition of acceleration (Eq. 2), defined and published by Newton, as the rate of change of velocity with time (or derivative of velocity with respect to time) and velocity as the rate of change of distance with time as given in equations (2) and (3) below,

$$a = \frac{dv}{dt} \cdot \tag{2}$$

$$v = \frac{ds}{dt} , \tag{3}$$

where s is the distance, v is the velocity, a is the acceleration, and t is time. Note that *ds, dv,* and *dt* are infinitesimal intervals of space, velocity, and time respectively. Here, we see that to measure velocity, one needs to have accurate measurements of small intervals of space and time.

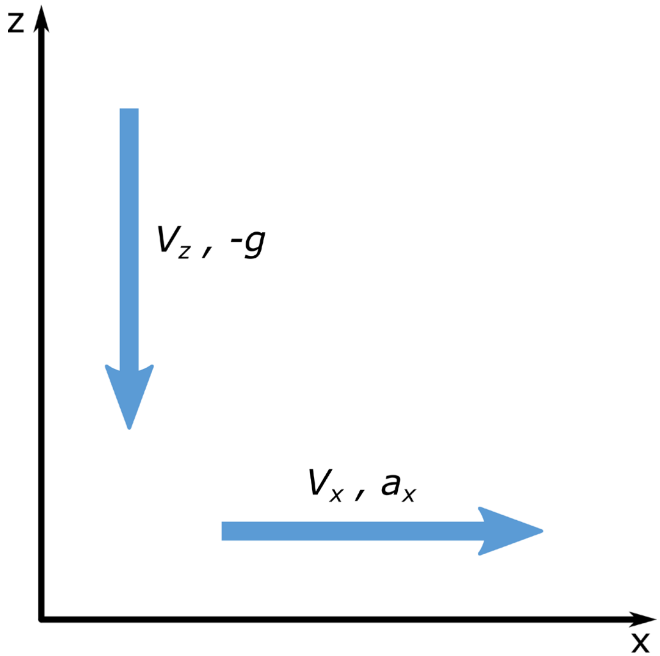

FIGURE 5.4. Leonardo da Vinci, Coordinate system and convention used in deriving equations of motion. a_x, a_y, and g are components of acceleration in x, y, and z directions, respectively. Similarly, v_z is the z-component of falling velocity vector in equation.

For constant acceleration and for objects that start to move from rest, equations of motion for distance traveled by an object under constant acceleration can be derived from equations (2) and (3) as

$$s = \frac{1}{2}at^2 . \tag{4}$$

For falling objects, gravitational acceleration commonly denoted as "*g*" will replace "*a*" in equation (4). However, as a vector quantity, acceleration and velocity can be expressed in terms of their vertical (z) and horizontal (x) components. Conventionally, gravitational acceleration is aligned with (–z), which means that it is oriented toward the earth (see figure 5.4)
Also, we can write equation (1) as

$$\upsilon_x = \upsilon_{x0} + a_x t(a), \upsilon_z = \upsilon_{z0} - gt(b) , \tag{5}$$

where v_{zo} and v_{xo} are initial velocities.

Finally, one can break down equation (4) into two components along the x and z directions as follows,

$$x = \upsilon_{x0}t + \frac{1}{2}a_x t^2 (a), z = \upsilon_{z0}t - \frac{1}{2}gt^2 (b) . \tag{6}$$

Equation (5b) reveals that a falling object's velocity has a linear relationship with time and is changing by a factor of "*g*" (approximately 9.8 m/s) with each consecutive second. For falling objects, equation (6b) says that these objects increase their falling distance as the square of time weighted by the gravitational factor "*g*." For projectiles, the ratio of initial launch velocities will define the launch angle, Θ. The equations of motion mentioned above can be combined to eliminate the time parameter to obtain an equation of motion (Eq. 7) that would represent the trajectory of any falling objects or projectiles in the absence of air resistance.

$$y = xtan(\theta) - \frac{gx^2}{2v^2 cos^2\theta} \qquad (7)$$

Depending on the initial velocity and launch angle (Θ), this equation (which belongs to the family of parabolic equations) would represent parabolas of different heights and widths.

DILEMMA OF LEONARDO'S PARABOLAS

Leonardo was fascinated by the nature of gravity. Almost two hundred years before Newton, he postulated that "every weight tends to fall towards the center [of the earth] by the shortest possible way."[34] He tried to decipher the secrets of gravitational law by conducting a series of simple but ingenious experiments, imagined and real. For example, he predicted parabolic trajectories for projectiles launched with various angles and initial velocities. In a plot (figure 5.5) in *Codex Madrid I*, folio 147 recto,[35] Leonardo suggested parabolic trajectories for a range of launch angles from 90 to near zero degrees.

This depiction by Leonardo is perhaps the first documented conjecture on the parabolic nature of ballistic trajectories, a discovery that is also commonly attributed to Galileo. To better understand the importance of the information presented in this drawing, we should compare them to trajectories that the equations of motion (Eq. 7) predict for the same set of conditions. As we have mentioned before, equation (7) belongs to a family of equations known as parabolic equations for which their solutions represent parabolas of different heights and widths. Figure 5.6 shows such trajectories calculated from equation (7) for projectiles launched with fixed initial velocity of *v* and for a wide range of launch angles under conditions close to those that Leonardo presents in his drawing (fig. 5.6). The reader may appreciate the striking similarities for trajectories launched at high and low angles. His parabolas fall short of the maximum ranges that exact parabolic curves reach for mid-launch angles. It is remarkable that he comes this close to drawing perfect parabolic shapes without having a quantitative knowledge of parabolic equations.

One may conclude that Leonardo had used his photographic memory in registering trajectories of these projectiles during an actual experiment. The dilemma

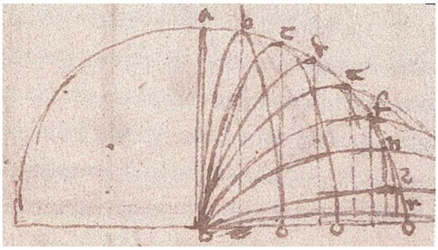

FIGURE 5.5. Leonardo da Vinci, Prediction of parabolic ballistic trajectories, Codex Madrid I, folio 147r, National Library of Spain. *(Note that we have transformed the image from its original mirror-imaged version to make it easier to read.)*

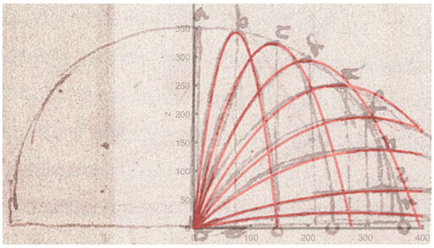

FIGURE 5.6. A direct-overlay comparison of trajectories calculated from equation (7) with those depicted by Leonardo for ballistic trajectories for various launch angles. Calculations are based on the author's best angles that could be estimated from Leonardo's drawings.

here is that, as we have indicated before, equation (7) is valid only when air resistance is not considered, and thus trajectories obtained through this equation are valid only when projectiles are launched under vacuum conditions or when resistance caused by airflow is small and of frictional nature. For almost all of the ballistic projectiles in Leonardo's time, the main airflow resistance must have been of the pressure type. Such projectiles do not follow a parabolic trajectory. As a matter of fact, equation (7) loses its parabolic characteristics when air-pressure resistance known as "form drag" is considered in deriving ballistic trajectories. Bertoloni-Meli notes that ballistic trajectories observed and reported by Galileo and others do not show parabolic behavior.[36] Some authors have suggested that Leonardo, raised under the influence of the classical approach of Aristotelian mechanics, might have used trajectories of issuing water jets to draw an analogy for the trajectory of ballistic projectiles. In Ms. C, folio 7 recto[37] (figure 5.5), Leonardo presents his observations of water jets issuing into the air from a water bag. While impressively accurate, trajectories depicted in Ms. C, folio 7 recto cannot be considered parabolic in general to justify Leonardo's nearly perfect rendering of ballistic trajectories in Codex Madrid I, folio 147 recto. This contradiction is intriguing and prompts us to question methods that Leonardo had used to deduce the parabolic trajectory concept. Did he comprehend and understand the nature of equations of motion well enough to draw nearly perfect parabolas as their solutions?

It would be intriguing to question whether Leonardo had the equations of motion unformulated in front of him (as Martin Kemp puts it) and whether he had the power of insight or feeling for these equations. We had to wait an additional two hundred years to see the power of differential calculus that allowed Newton and Leibnitz to formulate equations of motion as we know them today in order to correctly predict parabolic trajectories of ballistic projectiles. For Leonardo, waiting for the future of mathematics to arrive was not an option; that is why he refers to his observations of motion of objects as "equation di moti" or equations of motion.[38] In the history of physics and mechanics, the next time that we see the use of this phrase is by Sir Isaac Newton (1688) in his seventeenth-century book *Philosophiae naturalis principia mathematica*.[39]

LEONARDO AND FUTURISM

Was Leonardo a futurist? Do his unparalleled observations and deductions in the areas of cardiac anatomy and mathematics indicate that he was predicting the future? A gifted polymath and artist, Leonardo was not a science-fiction writer; nor was he intent on predicting the future. On the contrary, Leonardo believed in his ideas and hoped that his inventions might change lives in their own day. What is the difference then between an inventor from the fifteenth century and a science-fiction writer? The inventor is present, while the science-fiction creator projects and imagines a distant world. While Leonardo da Vinci anticipated many of the great scientific discoveries ahead of his time, nowhere did Leonardo discuss futurity. Instead, he embraced his present; he believed in the now. He so believed in the power of creating in the present that he took his ideas, ideas that would be enhanced later by individuals such as Copernicus, Galileo, and Newton, and turned his principles into first attempts at real applications, from calculators to helicopters, hydrodynamics to solar power. Leonardo was not waiting for the future to unfold and bring his creations to fruition; he was intent on attempting to create them during his own lifetime. In other words, he was more in line with Renaissance philosophers such as Michel de Montaigne, who warned against the "folly of gaping Mankind always after futurity . . . ," advising "us to lay hold of good which is present." He described "the most universal of human error": "For we are never present with, but always beyond ourselves."[40] Curiously, had Leonardo's ideas been published in his day, perhaps some of these modern advances in cardiology, mathematics, or physics would have been discovered centuries earlier. If there is one thing to which Leonardo's incredible array of notebooks attests, it is his very real presence as a man of the Early Modern era.